Problem 2.29 A 50-MHz generator with $Z_g = 50\ \Omega$ is connected to a load $Z_L = (50 - j25)\ \Omega$. The time-average power transferred from the generator into the load is maximum when $Z_g = Z_L^*$, where Z_L^* is the complex conjugate of Z_L. To achieve this condition without changing Z_g, the effective load impedance can be modified by adding an open-circuited line in series with Z_L, as shown in Fig. 2-40 (P2.29). If the line's $Z_0 = 100\ \Omega$, determine the shortest length of line (in wavelengths) necessary for satisfying the maximum-power-transfer condition.

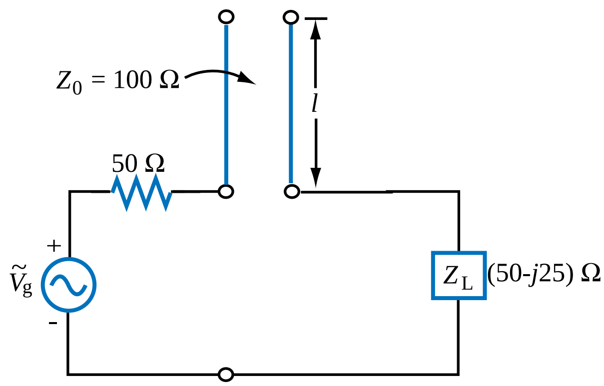

Figure P2.29: Transmission-line arrangement for Problem 2.29.

Solution: Since the real part of Z_L is equal to Z_g, our task is to find l such that the input impedance of the line is $Z_{in} = +j25\ \Omega$, thereby cancelling the imaginary part of Z_L (once Z_L and the input impedance the line are added in series). Hence, using Eq. (2.73),

$$-j100 \cot \beta l = j25,$$

or

$$\cot \beta l = -\frac{25}{100} = -0.25,$$

which leads to

$$\beta l = -1.326 \text{ or } 1.816.$$

Since l cannot be negative, the first solution is discarded. The second solution leads to

$$l = \frac{1.816}{\beta} = \frac{1.816}{(2\pi/\lambda)} = 0.29\lambda.$$