Problem 4.5 Find the total charge on a circular disk defined by $r \leq a$ and $z = 0$ if:

(c) $\rho_s = \rho_{s0} e^{-r}$ (C/m^2),

where ρ_{s0} is a constant.

Solution:

(c)

$$\begin{aligned} Q = \int_{r=0}^{a} \int_{\phi=0}^{2\pi} \rho_{s0} e^{-r} r \, dr \, d\phi &= 2\pi\rho_{s0} \int_0^a re^{-r} \, dr \\ &= 2\pi\rho_{s0} \left[-re^{-r} - e^{-r} \right]_0^a \\ &= 2\pi\rho_{s0} [1 - e^{-a}(1+a)]. \end{aligned}$$