Problem 4.17 Three infinite lines of charge, all parallel to the z-axis, are located at the three corners of the kite-shaped arrangement shown in Fig. 4-29 (P4.17). If the two right triangles are symmetrical and of equal corresponding sides, show that the electric field is zero at the origin.

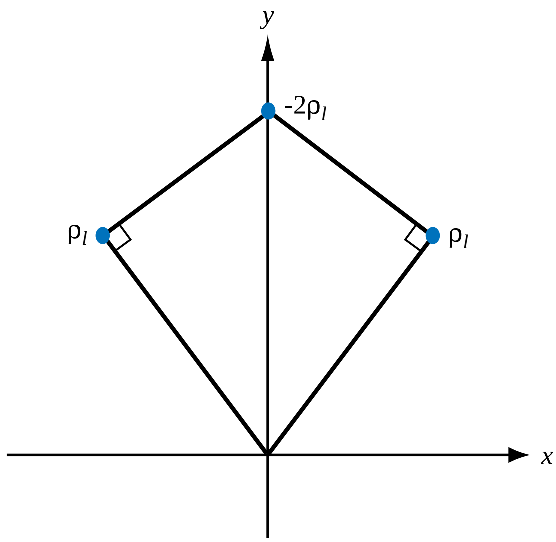

Figure P4.17: Kite-shaped arrangment of line charges for Problem 4.17.

Solution: The field due to an infinite line of charge is given by Eq. (4.33). In the present case, the total $\mathbf{E}$ at the origin is

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3.$$

The components of $\mathbf{E}_1$ and $\mathbf{E}_2$ along $\hat{\mathbf{x}}$ cancel and their components along $-\hat{\mathbf{y}}$ add. Also, $\mathbf{E}_3$ is along $\hat{\mathbf{y}}$ because the line charge on the y-axis is negative. Hence,

$$\mathbf{E} = -\hat{\mathbf{y}}\,\frac{2\rho_l\cos\theta}{2\pi\varepsilon_0 R_1} + \hat{\mathbf{y}}\,\frac{2\rho_l}{2\pi\varepsilon_0 R_2}\,.$$

But $\cos\theta = R_1/R_2$. Hence,

$$\mathbf{E} = -\hat{\mathbf{y}}\,\frac{\rho_l}{\pi\varepsilon_0 R_1}\,\frac{R_1}{R_2} + \hat{\mathbf{y}}\,\frac{\rho_l}{\pi\varepsilon_0 R_2} = 0.$$