Problem 4.31 Find the electric potential V at a location a distance b from the origin in the x–y plane due to a line charge with charge density ρ_l and of length l. The line charge is coincident with the z-axis and extends from $z = -l/2$ to $z = l/2$.
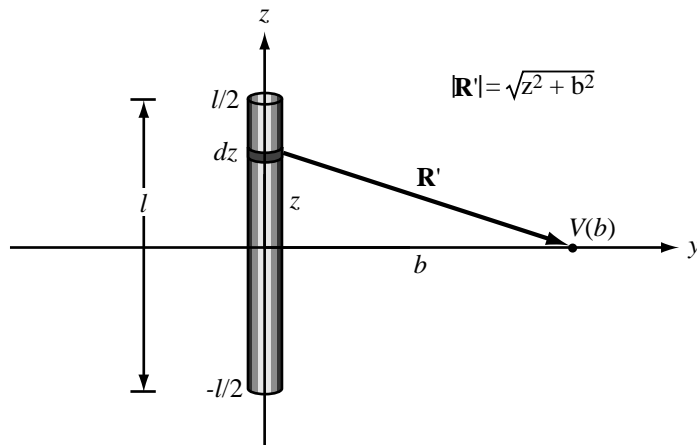

Figure P4.31: Line of charge of length ℓ.

Solution: From Eq. (4.48c), we can find the voltage at a distance b away from a line of charge [Fig. P4.31]:

$$V(b) = \frac{1}{4\pi\varepsilon} \int_{l'} \frac{\rho_l}{R'}\, dl' = \frac{\rho_l}{4\pi\varepsilon} \int_{-l/2}^{l/2} \frac{dz}{\sqrt{z^2 + b^2}} = \frac{\rho_l}{4\pi\varepsilon} \ln\left(\frac{l + \sqrt{l^2 + 4b^2}}{-l + \sqrt{l^2 + 4b^2}} \right).$$