Problem 5.33 The plane boundary defined by $z = 0$ separates air from a block of iron. If $\mathbf{B}_1 = \hat{\mathbf{x}}4 - \hat{\mathbf{y}}6 + \hat{\mathbf{z}}8$ in air ($z \geq 0$), find $\mathbf{B}_2$ in iron ($z \leq 0$), given that $\mu = 5000\mu_0$ for iron.

Solution: From Eq. (5.2),

$$\mathbf{H}_1 = \frac{\mathbf{B}_1}{\mu_1} = \frac{1}{\mu_1}(\hat{\mathbf{x}}4 - \hat{\mathbf{y}}6 + \hat{\mathbf{z}}8).$$

The z component is the normal component to the boundary at $z = 0$. Therefore, from Eq. (5.79), $B_{2z} = B_{1z} = 8$ while, from Eq. (5.85),

$$H_{2x} = H_{1x} = \frac{1}{\mu_1}4, \qquad H_{2y} = H_{1y} = -\frac{1}{\mu_1}6,$$

or

$$B_{2x} = \mu_2 H_{2x} = \frac{\mu_2}{\mu_1}4, \qquad B_{2y} = \mu_2 H_{2y} = -\frac{\mu_2}{\mu_1}6,$$

where $\mu_2/\mu_1 = \mu_r = 5000$. Therefore,

$$\mathbf{B}_2 = \hat{\mathbf{x}}20000 - \hat{\mathbf{y}}30000 + \hat{\mathbf{z}}8.$$