Problem 6.7 The rectangular conducting loop shown in Fig. 6-20 (P6.7) rotates at 6,000 revolutions per minute in a uniform magnetic flux density given by

$$\mathbf{B} = \hat{\mathbf{y}} 50 \quad (\mathrm{mT}).$$

Determine the current induced in the loop if its internal resistance is 0.5 Ω .

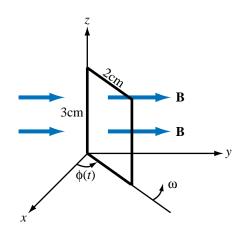


Figure P6.7: Rotating loop in a magnetic field (Problem 6.7).

Solution:

$$\begin{split} \Phi &= \int_{S} \mathbf{B} \cdot d\mathbf{S} = \hat{\mathbf{y}} 50 \times 10^{-3} \cdot \hat{\mathbf{y}} (2 \times 3 \times 10^{-4}) \cos \phi(t) = 3 \times 10^{-5} \cos \phi(t), \\ \phi(t) &= \omega t = \frac{2\pi \times 6 \times 10^{3}}{60} t = 200\pi t \quad (\text{rad/s}), \\ \Phi &= 3 \times 10^{-5} \cos(200\pi t) \quad (\text{Wb}), \\ V_{\text{emf}} &= -\frac{d\Phi}{dt} = 3 \times 10^{-5} \times 200\pi \sin(200\pi t) = 18.85 \times 10^{-3} \sin(200\pi t) \quad (\text{V}), \\ I_{\text{ind}} &= \frac{V_{\text{emf}}}{0.5} = 37.7 \sin(200\pi t) \quad (\text{mA}). \end{split}$$

The direction of the current is CW (if looking at it along $-\hat{\mathbf{x}}$ -direction) when the loop is in the first quadrant ($0 \le \phi \le \pi/2$). The current reverses direction in the second quadrant, and reverses again every quadrant.