Problem 6.13 The circular disk shown in Fig. 6-24 (P6.13) lies in the x–y plane and rotates with uniform angular velocity ω about the z-axis. The disk is of radius a and is present in a uniform magnetic flux density $\mathbf{B} = \hat{\mathbf{z}}B_0$. Obtain an expression for the emf induced at the rim relative to the center of the disk.

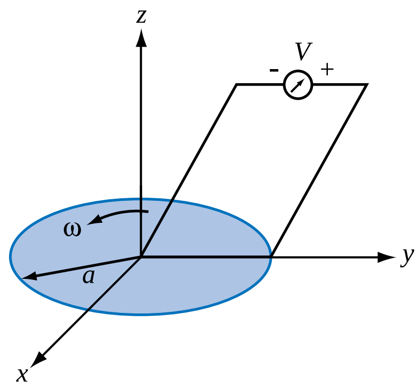

Figure P6.13: Rotating circular disk in a magnetic field (Problem 6.13).

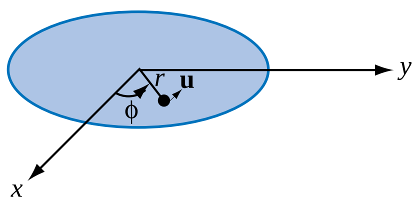

Figure P6.13: (a) Velocity vector **u**.

Solution: At a radial distance r, the velocity is

$$\mathbf{u} = \hat{\boldsymbol{\phi}}\,\omega r$$

where ϕ is the angle in the x–y plane shown in the figure. The induced voltage is

$$V = \int_0^a (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l} = \int_0^a [(\hat{\boldsymbol{\phi}}\,\omega r) \times \hat{\mathbf{z}}B_0] \cdot \hat{\mathbf{r}}\, dr.$$

$\hat{\boldsymbol{\phi}} \times \hat{\mathbf{z}}$ is along $\hat{\mathbf{r}}$. Hence,

$$V = \omega B_0 \int_0^a r\, dr = \frac{\omega B_0 a^2}{2}.$$