Problem 8.3 A plane wave traveling in a medium with $\varepsilon_{r_1} = 9$ is normally incident upon a second medium with $\varepsilon_{r_2} = 4$. Both media are made of nonmagnetic, non-conducting materials. If the magnetic field of the incident plane wave is given by

$$\mathbf{H}^{i} = \hat{\mathbf{z}}\, 2\cos(2\pi \times 10^9 t - ky) \quad \text{(A/m)},$$

(a) obtain time domain expressions for the electric and magnetic fields in each of the two media, and

(b) determine the average power densities of the incident, reflected and transmitted waves.

Solution:

(a) In medium 1,

$$u_p = \frac{c}{\sqrt{\varepsilon_{r_1}}} = \frac{3\times 10^8}{\sqrt{9}} = 1\times 10^8 \quad \text{(m/s)},$$

$$k_1 = \frac{\omega}{u_p} = \frac{2\pi\times 10^9}{1\times 10^8} = 20\pi \quad \text{(rad/m)},$$

$$\mathbf{H}^{i} = \hat{\mathbf{z}}\, 2\cos(2\pi\times 10^9 t - 20\pi y) \quad \text{(A/m)},$$

$$\eta_1 = \frac{\eta_0}{\sqrt{\varepsilon_{r_1}}} = \frac{377}{3} = 125.67\ \Omega,$$

$$\eta_2 = \frac{\eta_0}{\sqrt{\varepsilon_{r_2}}} = \frac{377}{2} = 188.5\ \Omega,$$

$$\begin{aligned}\mathbf{E}^{i} &= -\hat{\mathbf{x}}\, 2\eta_1 \cos(2\pi\times 10^9 t - 20\pi y)\\ &= -\hat{\mathbf{x}}\, 251.34\cos(2\pi\times 10^9 t - 20\pi y) \quad \text{(V/m)},\end{aligned}$$

$$\Gamma = \frac{\eta_2-\eta_1}{\eta_2+\eta_1} = \frac{188.5-125.67}{188.5+125.67} = 0.2,$$

$$\tau = 1+\Gamma = 1.2,$$

$$\begin{aligned}\mathbf{E}^{r} &= -\hat{\mathbf{x}}\, 251.34\times 0.2\cos(2\pi\times 10^9 t + 20\pi y)\\ &= -\hat{\mathbf{x}}\, 50.27\cos(2\pi\times 10^9 t + 20\pi y) \quad \text{(V/m)},\end{aligned}$$

$$\begin{aligned}\mathbf{H}^{r} &= -\hat{\mathbf{z}}\, \frac{50.27}{\eta_1}\cos(2\pi\times 10^9 t + 20\pi y)\\ &= -\hat{\mathbf{z}}\, 0.4\cos(2\pi\times 10^9 t + 20\pi y) \quad \text{(A/m)},\end{aligned}$$

$$\begin{aligned}\mathbf{E}_1 &= \mathbf{E}^{i} + \mathbf{E}^{r}\\ &= -\hat{\mathbf{x}}[25.134\cos(2\pi\times 10^9 t - 20\pi y) + 50.27\cos(2\pi\times 10^9 t + 20\pi y)] \quad \text{(V/m)},\end{aligned}$$

$$\mathbf{H}_1 = \mathbf{H}^{i} + \mathbf{H}^{r} = \hat{\mathbf{z}}[2\cos(2\pi\times 10^9 t - 20\pi y) - 0.4\cos(2\pi\times 10^9 t + 20\pi y)] \quad \text{(A/m)}.$$

In medium 2,

$$k_2 = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\, k_1 = \sqrt{\frac{4}{9}} \times 20\pi = \frac{40\pi}{3} \quad \text{(rad/m)},$$

$$\begin{aligned}\mathbf{E}_2 = \mathbf{E}^{\rm t} &= -\hat{\mathbf{x}}\, 251.34\, \tau \cos\left(2\pi \times 10^9 t - \frac{40\pi y}{3}\right) \\ &= -\hat{\mathbf{x}}\, 301.61 \cos\left(2\pi \times 10^9 t - \frac{40\pi y}{3}\right) \quad \text{(V/m)}, \\ \mathbf{H}_2 = \mathbf{H}^{\rm t} &= \hat{\mathbf{z}}\, \frac{301.61}{\eta_2} \cos\left(2\pi \times 10^9 t - \frac{40\pi y}{3}\right) \\ &= \hat{\mathbf{z}}\, 1.6 \cos\left(2\pi \times 10^9 t - \frac{40\pi y}{3}\right) \quad \text{(A/m)}.\end{aligned}$$

(b)

$$\begin{aligned}\mathbf{S}^{\rm i}_{\rm av} &= \hat{\mathbf{y}}\, \frac{|E_0|^2}{2\eta_1} = \hat{\mathbf{y}}\, \frac{(251.34)^2}{2 \times 125.67} = \hat{\mathbf{y}}\, 251.34 \quad \text{(W/m}^2\text{)}, \\ \mathbf{S}^{\rm r}_{\rm av} &= -\hat{\mathbf{y}}\, |\Gamma|^2 (251.34) = \hat{\mathbf{y}}\, 10.05 \quad \text{(W/m}^2\text{)}, \\ \mathbf{S}^{\rm t}_{\rm av} &= \hat{\mathbf{y}}\, (251.34 - 10.05) = \hat{\mathbf{y}}\, 241.29 \quad \text{(W/m}^2\text{)}.\end{aligned}$$