Problem 8.30 A perpendicularly polarized wave in air is obliquely incident upon a planar glass-air interface at an incidence angle of 30°. The wave frequency is 600 THz (1 THz = 10^{12} Hz), which corresponds to green light, and the index of refraction of the glass is 1.6. If the electric field amplitude of the incident wave is 50 V/m, determine

(a) the reflection and transmission coefficients, and

(b) the instantaneous expressions for **E** and **H** in the glass medium.

Solution:

(a) For nonmagnetic materials, $(\varepsilon_2/\varepsilon_1) = (n_2/n_1)^2$. Using this relation in Eq. (8.60) gives

$$\Gamma_\perp = \frac{\cos\theta_i - \sqrt{(n_2/n_1)^2 - \sin^2\theta_i}}{\cos\theta_i + \sqrt{(n_2/n_1)^2 - \sin^2\theta_i}} = \frac{\cos 30^\circ - \sqrt{(1.6)^2 - \sin^2 30^\circ}}{\cos 30^\circ + \sqrt{(1.6)^2 - \sin^2 30^\circ}} = -0.27,$$

$$\tau_\perp = 1 + \Gamma_\perp = 1 - 0.27 = 0.73.$$

(b) In the glass medium,

$$\sin\theta_t = \frac{\sin\theta_i}{n_2} = \frac{\sin 30^\circ}{1.6} = 0.31,$$

or $\theta_t = 18.21^\circ$.

$$\eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}} = \frac{\eta_0}{n_2} = \frac{120\pi}{1.6} = 75\pi = 235.62 \quad (\Omega),$$

$$k_2 = \frac{\omega}{u_p} = \frac{2\pi f}{c/n} = \frac{2\pi f n}{c} = \frac{2\pi \times 600 \times 10^{12} \times 1.6}{3 \times 10^8} = 6.4\pi \times 10^6 \text{ rad/m},$$

$$E_0^t = \tau_\perp E_0^i = 0.73 \times 50 = 36.5 \text{ V/m}.$$

From Eqs. (8.49c) and (8.49d),

$$\widetilde{\mathbf{E}}_\perp^t = \hat{\mathbf{y}} E_0^t e^{-jk_2(x\sin\theta_t + z\cos\theta_t)},$$

$$\widetilde{\mathbf{H}}_\perp^t = (-\hat{\mathbf{x}}\cos\theta_t + \hat{\mathbf{z}}\sin\theta_t)\frac{E_0^t}{\eta_2} e^{-jk_2(x\sin\theta_t + z\cos\theta_t)},$$

and the corresponding instantaneous expressions are:

$$\mathbf{E}_\perp^t(x,z,t) = \hat{\mathbf{y}} 36.5\cos(\omega t - k_2 x\sin\theta_t - k_2 z\cos\theta_t) \quad \text{(V/m)},$$

$$\mathbf{H}_\perp^t(x,z,t) = (-\hat{\mathbf{x}}\cos\theta_t - \hat{\mathbf{z}}\cos\theta_t) 0.16\cos(\omega t - k_2 x\sin\theta_t - k_2 z\cos\theta_t) \quad \text{(A/m)},$$

with $\omega = 2\pi \times 10^{15}$ rad/s and $k_2 = 6.4\pi \times 10^6$ rad/m.