Problem 3.22   Use the appropriate expression for the differential surface area $d\mathbf{s}$ to determine the area of each of the following surfaces:

(d) $R = 2$; $0 \le \theta \le \pi/3$; $0 \le \phi \le \pi$.

Also sketch the outlines of each of the surfaces.

Solution:
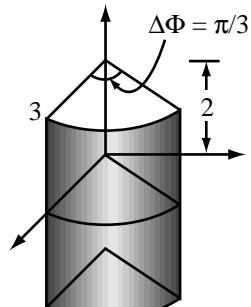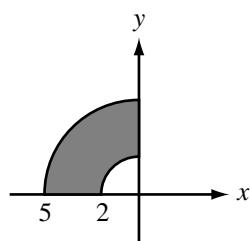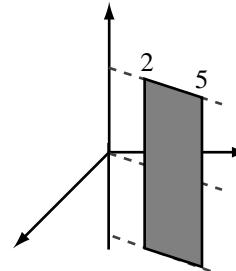
(a) (b) (c)
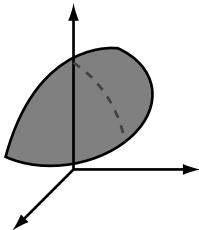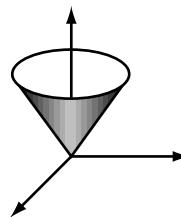
(d) (e)

Figure P3.22: Surfaces described by Problem 3.22.

(d) Using Eq. (3.50b),

$$A = \int_{\theta=0}^{\pi/3} \int_{\phi=0}^{\pi} \left(R^2 \sin\theta\right)\Big|_{R=2} d\phi\, d\theta = \left(\left(-4\phi\cos\theta\right)\big|_{\theta=0}^{\pi/3}\right)\Big|_{\phi=0}^{\pi} = 2\pi.$$