Problem 3.31   Transform the following vectors into spherical coordinates and then evaluate them at the indicated points:

(c) $\mathbf{C} = \hat{\mathbf{r}}\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi + \hat{\mathbf{z}}\cos\phi\sin\phi$ at $P_3(2, \pi/4, 2)$.

Solution: From Table 3-2:

(c)

$$\mathbf{C} = (\hat{\mathbf{R}}\sin\theta + \hat{\boldsymbol{\theta}}\cos\theta)\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi + (\hat{\mathbf{R}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta)\cos\phi\sin\phi$$

$$= \hat{\mathbf{R}}\cos\phi(\sin\theta + \cos\theta\sin\phi) + \hat{\boldsymbol{\theta}}\cos\phi(\cos\theta - \sin\theta\sin\phi) - \hat{\boldsymbol{\phi}}\sin\phi,$$

$$P_3 = \left(\sqrt{2^2 + 2^2}, \tan^{-1}(2/2), \pi/4\right) = (2\sqrt{2}, 45^\circ, 45^\circ),$$

$$\mathbf{C}(P_3) \approx \hat{\mathbf{R}}0.854 + \hat{\boldsymbol{\theta}}0.146 - \hat{\boldsymbol{\phi}}0.707.$$