Problem 9.3 Determine the (a) direction of maximum radiation, (b) directivity, (c) beam solid angle, and (d) half-power beamwidth in the *x*–*z* plane for an antenna whose normalized radiation intensity is given by

$$F(\theta,\phi) = \begin{cases} 1, & \text{for } 0 \le \theta \le 60^\circ \text{and } 0 \le \phi \le 2\pi, \\ 0, & \text{elsewhere.} \end{cases}$$

Suggestion: Sketch the pattern prior to calculating the desired quantities.

Solution: The direction of maximum radiation is a circular cone 120° wide centered around the $+\hat{\mathbf{z}}$-axis. From Eq. (9.23),

$$D = \frac{4\pi}{\iint_{4\pi} F\, d\Omega} = \frac{4\pi}{\int_0^{2\pi}\int_0^{60^\circ} \sin\theta\, d\theta\, d\phi} = \frac{4\pi}{2\pi(-\cos\theta)\big|_{0^\circ}^{60^\circ}} = \frac{2}{-\frac{1}{2}+1} = 4 = 6 \text{ dB},$$

$$\Omega_p = \frac{4\pi \text{ sr}}{D} = \frac{4\pi \text{ sr}}{4} = \pi \quad \text{(sr)}.$$

The half power beamwidth is $\beta = 120^\circ$.